2017 Trial Higher School Certificate Examination

Killara HIGH SCHOOL

MATHEMATICS

24 July 2017

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black pen.
- NESA approved calculators may be used.
- **Commence each new question in a new booklet.** Write on both sides of the paper.
- A reference sheet is provided.
- In Question 11–16 show relevant mathematical reasoning and/or calculations
- At the conclusion of the examination, bundle the booklets used in the correct order **including your reference sheet** within this paper and hand to examination supervisor.

Total Marks: 100

Section 1 – 10 marks (pages 3 - 7)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section 2 – 90 marks (pages 9 - 17)

- Attempt Questions 11 – 16
- Allow about 2 hours and 45 minutes for this section

NESA NUMBER: .... **# BOOKLETS USED**: .....

Marker's use only.

QUESTION	1-10	11	12	13	14	15	16	**Total**
MARKS	/10	/15	/15	/15	/15	/15	/15	/100

This task constitutes 40% of the HSC Course Assessment

Section I

10 marks
Attempt Question 1 to 10
Allow approximately 15 minutes for this section

Mark your answers on the answer grid provided (labelled as page 19).

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1. Evaluate $\sqrt{\dfrac{3^2 - 2^2}{10^2 \times 150}}$ to 3 significant figures.

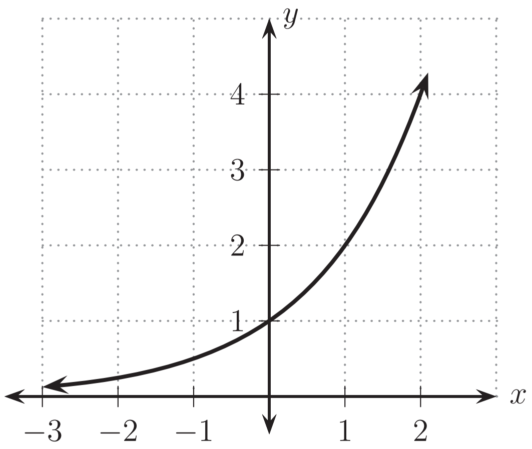
(A) 0.02

(B) 0.018

(C) 0.0182

(D) 0.0183

2. The graph of $y = 2^x$ is given below.



The solution to $2^x = 2x$ is

(A) $x = 0$

(B) $x = 1$

(C) $x = 1$ and $x = 2$

(D) $x = 2$ and $x = 4$

3. What is the domain of $y = \log_e(x+2) + 3$?

(A) $x > 2$

(B) $x > -2$

(C) $x > 3$

(D) $x > -3$

4. What is the angle of inclination of the straight line $x + 2y + 4 = 0$ to the nearest degree?

(A) $27°$

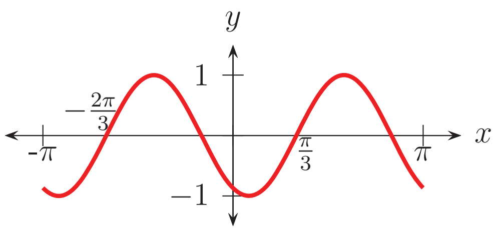
(B) $45°$

(C) $135°$

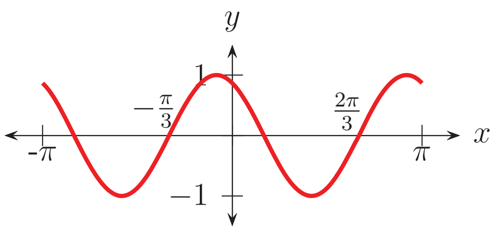
(D) $153°$

5. Which of the following shows the graph of $y = \sin\left(2x + \dfrac{2\pi}{3}\right)$?

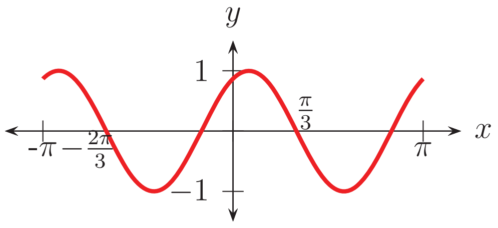
(A)



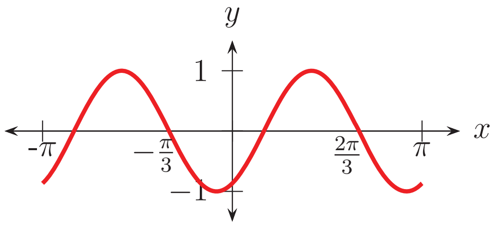
(C)



(B)



(D)



6. The primitive function of x^{-3} is:

(A) $\dfrac{x^{-2}}{2} + c$

(B) $\dfrac{x^{2}}{2} + c$

(C) $-\dfrac{x^{-2}}{2} + c$

(D) $-\dfrac{x^{2}}{2} + c$

7. The solutions to $cos x = \dfrac{\sqrt{3}}{2}$ for $-\pi \leq x \leq \pi$ are:

(A) $x = -\dfrac{\pi}{3}, \dfrac{\pi}{3}$

(B) $x = -\dfrac{2\pi}{3}, \dfrac{2\pi}{3}$

(C) $x = -\dfrac{\pi}{6}, \dfrac{\pi}{6}$

(D) $x = -\dfrac{5\pi}{6}, \dfrac{5\pi}{6}$

8. For what values of k does $3x^2 - kx + 3$ have no real roots?

(A) $k < 6$

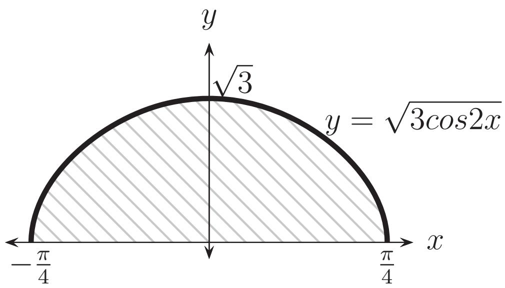
(B) $-6 < k < 6$

(C) $k < -6$

(D) $k < -6$ and $k > 6$

9. The diagram shows the region bounded by the curve $y = \sqrt{3\cos 2x}$ and the x- axis for $-\dfrac{\pi}{4} \leq x \leq \dfrac{\pi}{4}$

The region is rotated about the x axis to form a solid.



Which of the following gives the volume of the solid?

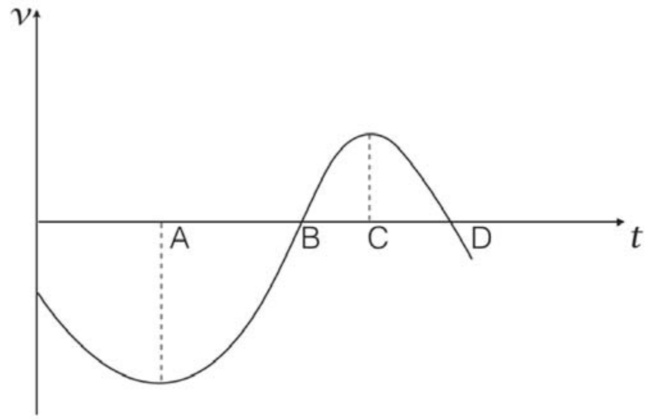
(A) $V = 3\pi \displaystyle\int_0^{\frac{\pi}{4}} \cos 2x \; dx$

(B) $V = 9\pi \displaystyle\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos 2x \; dx$

(C) $V = 6\pi \displaystyle\int_0^{\frac{\pi}{4}} \cos 2x \; dx$

(D) $V = 6\pi \displaystyle\int_0^{\frac{\pi}{4}} \cos 4x \; dx$

10. The diagram below shows the velocity-time graph for a particle moving in a straight line. When is the particle furthest from its initial position?



(A) Point A

(B) Point B

(C) Point C

(D) Point D

Section II

90 marks
Attempt Questions 11 to 16
Allow approximately 2 hours and 45 minutes for this section.

Write your answers in the writing booklets supplied. Additional writing booklets are available.
Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 Marks) Use a SEPARATE writing booklet **Marks**

(a) Simplify $\dfrac{3x}{x+2} - \dfrac{5x-19}{x^2+5x+6}$ **2**

(b) Solve $2^{3x-1} = \dfrac{1}{8}$ **2**

(c) If $\dfrac{2}{1+\sqrt{5}} = a + b\sqrt{5}$ find the values of a and b **2**

(d) The probability that an item is defective is 0.03. A batch of 600 items is examined. **1**
How many items from this batch are not defective?

(e) Differentiate $\dfrac{5-3x^2}{1+2x}$ with respect to x . **2**

(f) Solve $5 + 4x - x^2 \leq 0$. **2**

(g) Show that $\sec x \times \cot x = \operatorname{cosec} x$. **1**

(h) Find the equation of the tangent to the curve $y = (2x-3)^5$ at the point where $x = 1$. **3**

Question 12 (15 Marks) Use a SEPARATE writing booklet **Marks**

(a) The quadratic equation $x^2 - 3x + 10$ has roots α and β. **2**

Find $\alpha^2 + \beta^2$

(b) Find $\int \frac{x+1}{x^2+2x} dx$. **2**

(c) A boat leaves a port A and sails on a bearing of 130° for 10 km until it reaches a point B. It then heads on a bearing of 250° until it reaches a point C. The original port A is on a bearing of 20° from point C. **2**

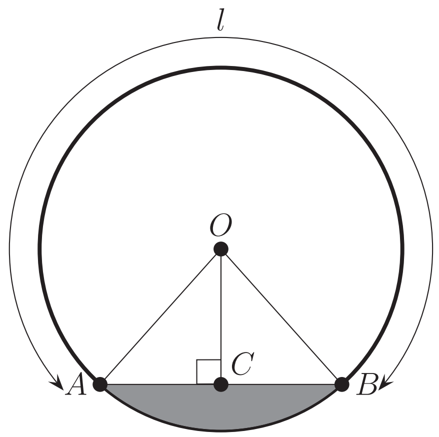
How far is the boat from the original port A?

(d) Find the values of A , B and C if **2**

$$3x^2 + 7x - 2 \equiv A(x+2)^2 + B(x-2) + C$$

(e) Find the derivative of $y = x^3 e^{x+5}$ with respect to x. **2**

(f) On the diagram below, O is the centre of the circle and the line OC makes an angle of 90 degrees with AB. The arc marked l is 11π cm long and the radius of the circle is 9 cm.



i. Prove that $\triangle AOC \equiv \triangle BOC$. **2**

ii. Find the reflex $\angle AOB$. Give your answer in radians. **1**

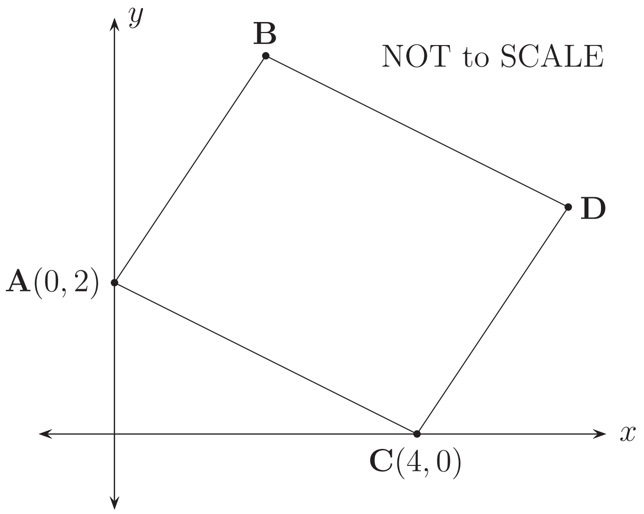
iii. Find the area of the shaded region. Give your answer correct to 2 decimal places. **2**

Question 13 (15 Marks) Use a SEPARATE writing booklet **Marks**

(a) The focus of a parabola is $(1, -4)$ and the directrix is $y = 0$. Find the vertex. **1**

(b) The parallelogram $ABCD$ has coordinates $A(0, 2)$ and $C(4, 0)$.

The line AB has the equation $3x - 2y + 4 = 0$ and the line BD has the equation $x + 2y - 12 = 0$.



i. Show that the point B has coordinates $(2, 5)$. **2**

ii. Find the perpendicular distance of B from AC. **2**

iii. Hence or otherwise find the area of the parallelogram $ABCD$. **2**

(c) Solve $|2x + 1| = 3x + 4$. **3**

(d) Jenna and Aishani are playing a game with a pack of cards. A standard pack of cards contains 52 cards and has 4 different suits.

Each player takes turns picking a card from the deck, the cards are NOT replaced. The winner of the game is the first player to pick a heart card from the deck. Jenna starts the game.

i. What is the probability that Jenna wins the game on her first pick? **1**

ii. What is the probability that Jenna wins the game on her second pick? **2**

(e) Find the equation of the locus of all points which are equidistant from the two parallel lines $3x - 2y + 4 = 0$ and $3x - 2y - 8 = 0$. **2**

Question 14 (15 Marks) Use a SEPARATE writing booklet **Marks**

(a) Sketch the region on the Cartesian plane for which the following inequalities hold true, showing all important features. **3**

$$y \leq x$$
$$y \geq 0$$
$$y \leq \sqrt{6 - x^2}$$

(b) Use Simpsons Rule with 3 function values to evaluate $\int_0^{\frac{\pi}{4}} \sec x \, dx$. **3**
Give your answer correct to 2 decimal places .

(c) A particle moves in a straight line so that its velocity v metres per second at time t is given by $v = 4 - \dfrac{2}{t+1}$. Initially the particle is at the origin.

i. Find an expression for the position x of the particle at any time t. **2**

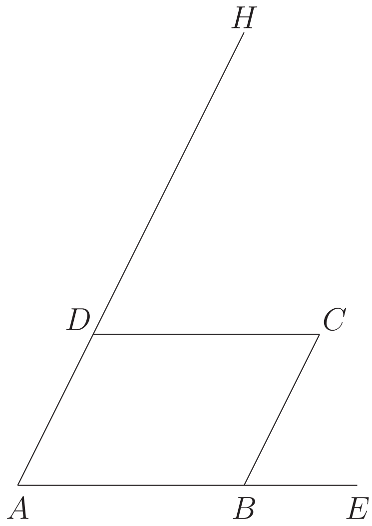
ii. Explain why the velocity of the particle is never 4 ms^{-1}. **1**

iii. Find the acceleration of the particle when $t = 2$ seconds . **2**

Question 14 continues on page 11

Question 14 continued

(d) In a parallelogram $ABCD$ the sides AB and AD are extended to E and H respectively so that $\dfrac{AD}{DH} = \dfrac{BE}{AB}$.



Copy or trace the diagram into your writing booklet .

i. Prove that $\triangle HDC$ and $\triangle CBE$ are similar. **3**

ii. Hence or otherwise show that the points H, C and E lie on a straight line . **1**

Question 15 (15 Marks) Use a SEPARATE writing booklet **Marks**

(a) Consider the curve given by $y = 1 - 12x + x^3$ for $-4 \leq x \leq 4$.

i. Find the stationary points and determine their nature. **3**

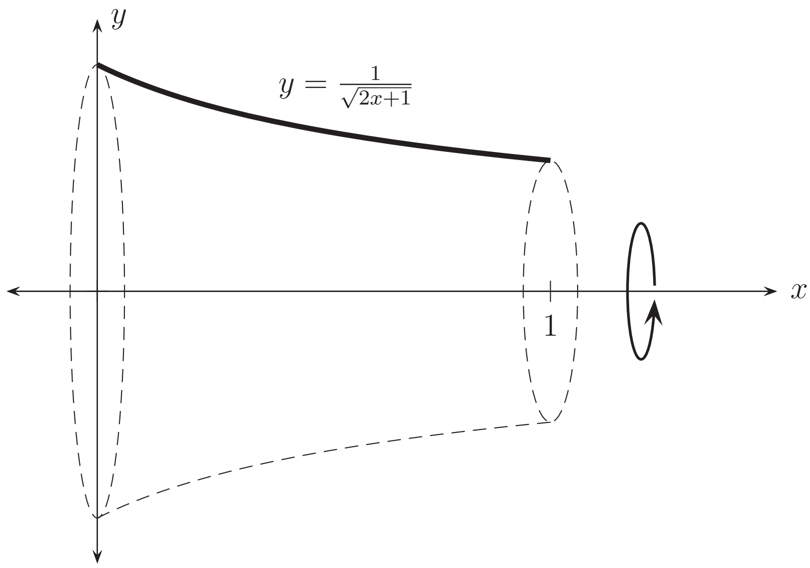
ii. Find the point of inflexion. **2**

iii. Sketch the curve for $-4 \leq x \leq 4$. **1**

(b) i. Differentiate $y = \sqrt{9 - x^2}$ with respect to x **2**

ii. Hence or otherwise find $\displaystyle\int \frac{6x}{\sqrt{9 - x^2}}\, dx$. **2**

(c) The part of the curve $y = \dfrac{1}{\sqrt{2x+1}}$ between $x = 0$ and $x = 1$ is rotated around the x axis. **3**

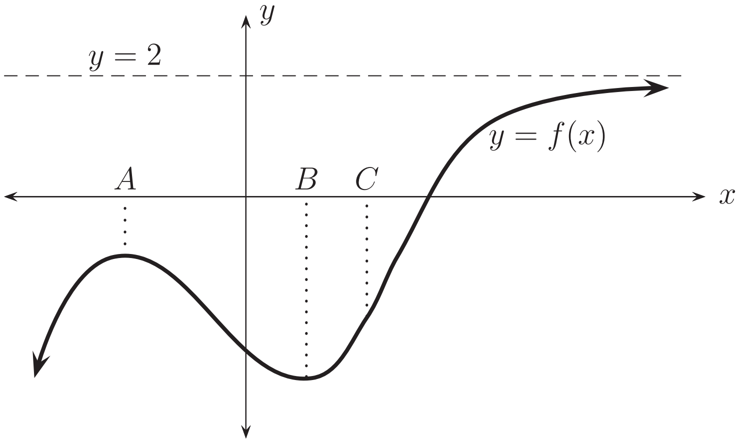


Find the exact volume of the solid obtained.

Question 15 continues on page 12

Question 15 continued

(d) Given the graph of $y = f(x)$ below, sketch the graph of $f'(x)$ clearly showing the points A, B and C on your graph. **2**



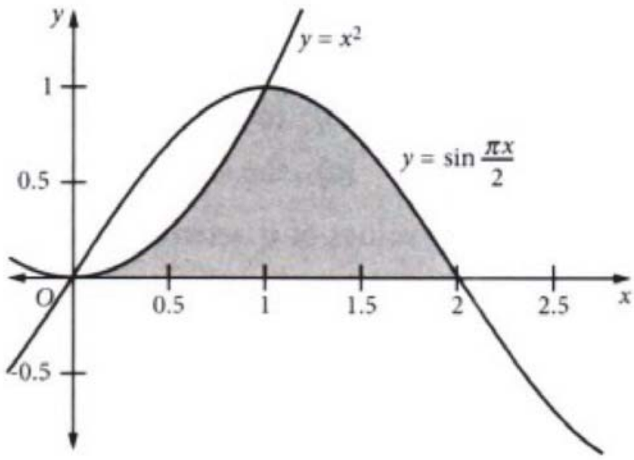
Question 16 (15 Marks) Use a SEPARATE writing booklet **Marks**

(a) A full petrol tank is punctured by a rock. The amount of fuel left in the tank at a time t after the puncture occurred can be modelled by the equation $Q = Ae^{-kt}$.

i. If $\frac{5}{8}$ of the fuel is left in the tank 10 minutes after the puncture, determine what fraction of the fuel will be left in the tank 20 minutes after the puncture. **3**

ii. Given the capacity of the fuel tank is 60 litres, at what rate (in litres per minute) is the fuel flowing out of the tank 20 minutes after the puncture? Give your answer correct to one decimal place. **1**

(b) The shaded region in the diagram is bounded by the curves $y = \sin\frac{\pi x}{2}$, $y = x^2$ and the x-axis.



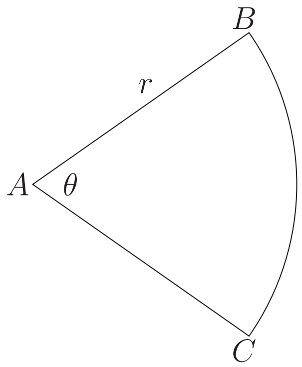
i. Given that the two curves intersect at $x = 1$, calculate the exact area of the shaded region. **2**

ii. Write an expression that will give the volume of the solid of revolution that is formed when the shaded region is rotated about the x-axis. **Do NOT evaluate your expression**. **1**

Question 16 continues on page 15

Question 16 continued

(c) In the figure AB and AC are radii of length r metres of a circle with centre A. The arc BC subtends an angle θ radians at A.



The perimeter of the above figure is 8 metres.

i. Show that the area of the sector ABC is given by $\dfrac{32\theta}{(2+\theta)^2}$ **1**

ii. Hence or otherwise show that the maximum area of the sector is 4 square metres **3**

(d) An open box is to be made from thin sheet metal with a square base having edges x metres long and vertical sides y metres long. It is to have a volume of $2m^3$.

i. Show that the inner surface area of the box (assuming negligible thickness) is given by $A = \left(\dfrac{8}{x} + x^2\right)$ **1**

ii. Prove that the inner surface area of the box will be a minimum if $x = 2y$. **3**

End of Examination ☺

2017 Trial Higher School Certificate Examination

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MATHEMATICS

NESA NUMBER: Answers

Section 1 –Multiple Choice Answer Sheet

Use this multiple choice answer sheet for Questions 1 - 10. Detach this sheet.

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ☒ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

A ☒ B ☒ (correct) C ○ D ○

Start Here →

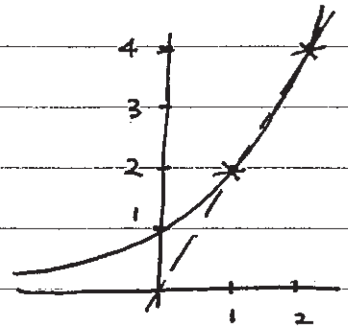
	A	B	C	D
1.	○	○	○	●
2.	○	○	●	○
3.	○	●	○	○
4.	○	○	○	●
5.	○	○	●	○
6.	○	○	●	○
7.	○	○	●	○
8.	○	●	○	○
9.	○	○	●	○
10.	○	●	○	○

Yr 12 - 2unit Trial

worked multiple choice answers

1. $0.0182|\textcircled{5}7$

$= 0.0183$ (D)

2. 

$\therefore x = 1, x = 2$ (C)

3. domain of $\ln(x+2)$ note → +3 not important

$x + 2 > 0$

$x > -2$ (B) as ln of $x \le 0$ undefined

4. $x + 2y + 4 = 0$

$y = -\frac{1}{2}x - 2$

$\tan^{-1}\left(-\frac{1}{2}\right) = 153°$ (D)

5. need to find when $2x + \frac{2\pi}{3} = 0$

$2x = -\frac{2\pi}{3}$

$x = -\frac{\pi}{3}$

so we need the graph where the sine curve starts at $x = -\frac{\pi}{3}$

$\therefore$ (C)

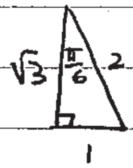
6 $\int x^{-3}$

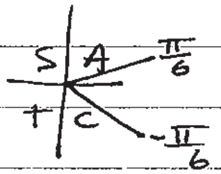
$= \dfrac{x^{-2}}{-2} + c$

$= \dfrac{-x^{-2}}{2} + c$

$\therefore$ (C)

7 $\cos x = \dfrac{\sqrt{3}}{2}$





$\therefore x = \dfrac{\pi}{6}, \dfrac{-\pi}{6}$

$\therefore$ (C)

8 $3x^2 - kx + 3$

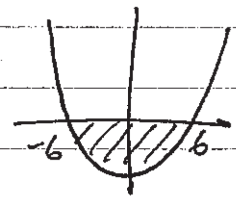
need $\Delta < 0$

$b^2 - 4ac < 0$

$(-k)^2 - 4.3.3 < 0$

$k^2 - 36 < 0$

$(k-6)(k+6) < 0$



$\therefore -6 < k < 6$

$\therefore$ (B)

9 $y = \sqrt{3\cos 2x}$

$$V = \pi \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} y^2 \, dx \quad \rightarrow \quad y^2 = 3\cos 2x$$

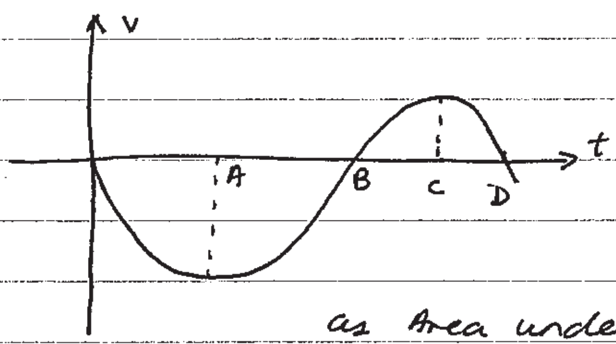
$$V = \pi \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 3\cos 2x \, dx$$

$$V = 3\pi \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos 2x \, dx$$

as area from 0 to $\frac{\pi}{4}$
equals area from 0 to $-\frac{\pi}{4}$

$$\therefore V = 6\pi \int_{0}^{\frac{\pi}{4}} \cos 2x \, dx$$

10



as Area under curve = displacement
and we aren't concerned with max distance →
or min distance ← we want furthest distance
then just need point that gives |max area|
∴ point B
∴ (B)

11 a $\dfrac{3x}{x+2} - \dfrac{5x-19}{x^2+5x+6}$

$= \dfrac{3x}{x+2} - \dfrac{5x-19}{(x+2)(x+3)}$

$= \dfrac{3x(x+3) - 5x + 19}{(x+2)(x+3)}$

$= \dfrac{9x^2+9x - 5x + 19}{(x+2)(x+3)}$

$= \dfrac{9x^2+4x+19}{(x+2)(x+3)}$

1 mark

Correctly identify and apply common factors.

Don't identify common factors but multiply and then group like terms

2 marks

Correct final expression.

This question caught a lot of students who failed to apply the negative to both terms in the numerator ie.

$- \dfrac{5x-19}{(x+2)(x+3)} = \dfrac{-5x+19}{(x+2)(x+3)} \neq \dfrac{-5x-19}{(x+2)(x+3)}$

b $2^{3x-1} = \frac{1}{8}$

$2^{3x-1} = 2^{-3}$

$3x-1 = -3$

$3x = -2$

$x = \frac{-2}{3}$

1 mark

~~Correctly~~ Used negative index to chang $\frac{1}{8}$ to 2^{-3}

or

Used logarithms without finalising expression to get $-\frac{2}{3}$

2 marks

correct solution

A lot of students used logarithms when they didn't have to and confused ~~themt~~ themselves.

11 c $\frac{2}{1+\sqrt{5}}$

$= \frac{2(1-\sqrt{5})}{(1+\sqrt{5})(1-\sqrt{5})}$

$= \frac{2-2\sqrt{5}}{-4}$

$= \frac{-1+\sqrt{5}}{2}$

$a = -\frac{1}{2} \quad b = \frac{1}{2}$

1 mark	
Correct rationalisation. No simplifying needed	Generally well done.
2 marks	
Inferred a & b correctly. Must have written as $a = -\frac{1}{2}$ $b = \frac{1}{2}$. $\frac{-1+\sqrt{5}}{2}$ does not count.	

1 d) P(not defective) = 0.97
Expected value is 0.97 × 600 = 582

1 mark Correct answer.	Generally well done

1 e) $\frac{d}{dx}\left(\frac{5-3x^2}{1+2x}\right) = \frac{-6x(1+2x) - 2(5-3x^2)}{(1+2x)^2}$

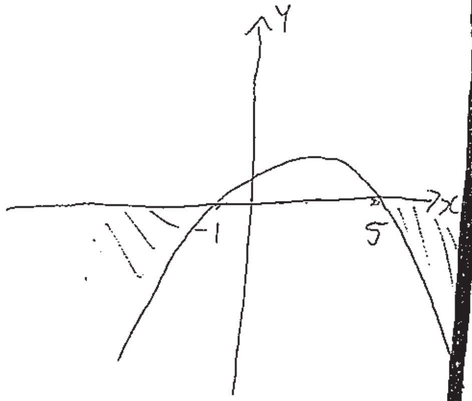
$\frac{d}{dx}\left(\frac{5-3x^2}{1+2x}\right) = \frac{-6x - 12x^2 - 10 + 6x^2}{(1+2x)^2}$

$= \frac{-6x^2 - 6x - 10}{(1+2x)^2}$

$= \frac{-2(3x^2+3x+5)}{(1+2x)^2}$

1 mark	
Correct use of quotient rule	Generally well done.
2 marks	
$\frac{-6x^2-6x-10}{(1+2x)^2}$	

1 f) $(x+5)(x+1) \geq 0$



$x \leq -1$

$x \geq 5$

1 mark
Correctly identified the boundaries

2 marks
Correct answer

Knowing how the boundary condition worked seemed problematic. Very few tested a point or drew a diagram.

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g) $LHS = \sec x \times \cot x$

$= \frac{1}{\cos x} \times \frac{\cos x}{\sin x}$

$= \frac{1}{\sin x} = \operatorname{cosec} x = RHS$

1 mark
Logical expression

Generally well done.

---

11 h) $y = (2x-3)^5$

$\frac{dy}{dx} = 5(2x-3)^4 \times 2$

$= 10(2x-3)^4$

at $x = 1$

$\frac{dy}{dx} = 10(1)^4$

$= 10$

at $x = 1$

$y = (2-3)^5$

$= -1$

$\therefore$ tangent is $y = 10x + c$

$-1 = 10 + c$

$c = -11$

Equation not [illegible] $y = 10x - 11$

1 mark
Correct differentiation or correct point

2 marks
Both

3 marks
Correct equation

Generally well done.

Question 12 (15 Marks)

(a) **1 mark** Finds correct expression for $\alpha+\beta$ and $\alpha\beta$
1 mark Finds correct expression for $\alpha^2+\beta^2$

$$\begin{aligned}\text{For the equation } x^2-3x+10&=0\\ \alpha+\beta&=3\\ \alpha\beta&=10\\ \text{So } \alpha^2+\beta^2&=(\alpha+\beta)^2-2\alpha\beta\\ \therefore \alpha^2+\beta^2&=(3)^2-2\times 10\\ &=-11\end{aligned}$$

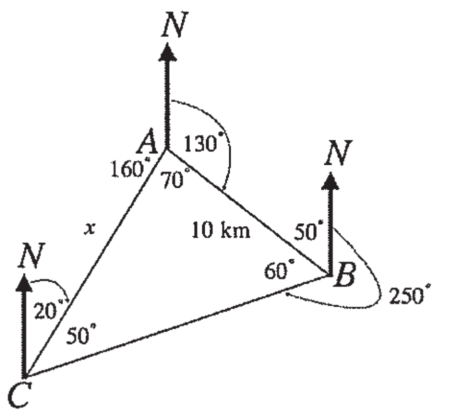
Markers Note Generally well done – students need to be careful of the signs

(b) **1 mark** Finds correct expression for making the standard integral with the $\frac{1}{2}$ and the numerator $2x+2$
1 mark Evaluates integral correctly

$$\begin{aligned}\int \frac{x+1}{x^2+2x}\,dx &= \frac{1}{2}\int \frac{2x+2}{x^2+2x}\,dx\\ &= \frac{1}{2}\log_e\left|x^2+2x\right|+C\end{aligned}$$

Markers Note All students need to be able to recognise the manipulations needed to evaluate standard integrals

(c) **1 mark** Correct use of sin rule for their diagram
1 mark Correct diagram leading to correct answer



$$\begin{aligned}\frac{x}{\sin 60}&=\frac{10}{\sin 50}\\ \therefore x&=\frac{10\times\sin 60}{\sin 50}\\ &=11.31 \text{ correct to 2 decimal places}\end{aligned}$$

Markers Note Students who drew big diagrams were more likely to succeed - this is a very standard bearing question and should be straightforward.

(d) **1 mark** Correct expansion of $A(x+2)^2 + B(x-2) + C$
1 mark Correct values for A, B and C

$$\begin{aligned} 3x^2 + 7x - 2 &\equiv A(x+2)^2 + B(x-2) + C \\ &= Ax^2 + 4Ax + 4A + Bx - 2B + C \\ &= Ax^2 + (4A+B)x + (4A - 2B + C) \end{aligned}$$

Equalting coefficients

$$\begin{aligned} A &= 3 \\ 4A + B &= 7 \\ \therefore B &= -5 \\ 4A - 2B + C &= -2 \\ \therefore C &= -24 \end{aligned}$$

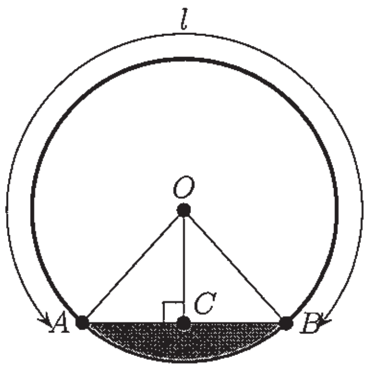
Markers Note Generally well done - students who included all steps were more likely to have success. Many minor algebraic manipulation errors. Substituting specific values of x often lead to errors.

(e) **2 marks** Identifying and correctly applying the product rule leading to the correct derivative (didn't need to be factorised)
1 mark Reasonable attempt to apply the product rule with only a minor error leading to an incorrect solution

$$\begin{aligned} y &= x^3 e^{x+5} \\ \therefore \frac{dy}{dx} &= (3x^2)(e^{x+5}) + (x^3)(e^{x+5}) \\ &= x^2 e^{x+5}(3+x) \end{aligned}$$

Markers Note Generally well done but some students are still not recognising when to use the product rule and not differentiating exponential functions corectly

(f)



i. **2 marks** Correct proof with all reasons stated

1 mark Reasonable attempt to apply the product rule with only a minor error leading to an incorrect solution

$$AO = OB \qquad \text{(equal radii of a circle)}$$
$$\angle OAC = \angle OCB = 90° \qquad \text{(given and straight angle)}$$
$$OC \text{ is common}$$
$$\therefore \triangle AOC \equiv \triangle BOC \qquad \text{(Right Angle, Hypotenuse, Side)}$$

Markers Note Generally well done but some students tried to use SAS without the included angle! Other proofs were less efficient.

ii. **1 mark** Correct solution with working

$$l = r\theta \qquad \text{(on the Reference Sheet)}$$
$$\therefore 11\pi = 9\theta$$
$$\therefore \theta = \frac{11\pi}{9}$$

Markers Note Generally well done although some gave solution as $\frac{7\pi}{9}$ which is NOT the reflex angle!

iii. **1 mark** Correct substitution into formula or other alternative method

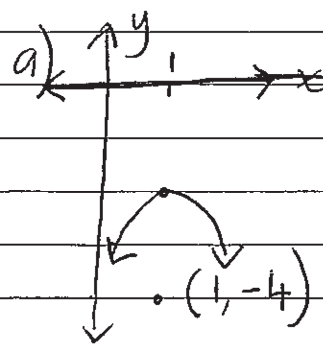
1 mark Correct answer to 2 decimal places or if error in substitution that still mathematically correct their answer

$$A = \frac{1}{2}r^2(\theta \sin\theta) \qquad \text{not on the reference sheet but useful!}$$
$$= \frac{1}{2}\left(\frac{7\pi}{9} - \sin\frac{7\pi}{9}\right)$$
$$= 72.93\ cm^2$$

Markers Note Many did not remember this formula - alternative methods had little success. Many did not use radians or made calculator errors.

Question 13

a)



focal length = 2

∴ vertex = (1, −2)

> 1 mark for stating the vertex

b) i) $3x - 2y + 4 = 0$
$x + 2y - 12 = 0$] solve simultaneously

$x = -2y + 12$

substitute into first equation

$3(-2y + 12) - 2y + 4 = 0$

$-6y + 36 - 2y + 4 = 0$

$-8y + 40 = 0$

$y = 5$

$x = -2(5) + 12$

$= 2$

Point of intersection = (2, 5)

> • 1 mark for working towards solving simultaneous equations
> • 2 marks for correctly solving

ii) Equation of line AC: $y = \frac{-x}{2} + 2$

$x + 2y - 4 = 0$

$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

$= \frac{|1(2) + 2(5) + -4|}{\sqrt{1^2 + 2^2}}$

$= \frac{8}{\sqrt{5}}$

> • 1 mark for correctly finding equation of line AC
> • 2 marks for correctly finding perpendicular distance

iii) distance $AC = \sqrt{2^2 + 4^2}$
$= \sqrt{20}$
$= 2\sqrt{5}$

Area $= \frac{8}{\sqrt{5}} \times 2\sqrt{5}$
$= 16$ units^2

- 1 mark for correctly finding distance AC
- 2 marks for correctly finding area

c) $|2x+1| = 3x+4$

Case 1

$2x+1 = 3x+4$
$x = -3$
test:
LHS $= |2(-3)+1|$
$= |-5|$
$= 5$
RHS $= 3(-3)+4$
$= -5$
$\neq$ LHS
Not a solution.

Case 2

$2x+1 = -3x-4$
$5x = -5$
$x = -1$
test:
LHS $= |2(-1)+1|$
$= |-1|$
$= 1$
RHS $= 3(-1)+4$
$= 1$
$=$ LHS

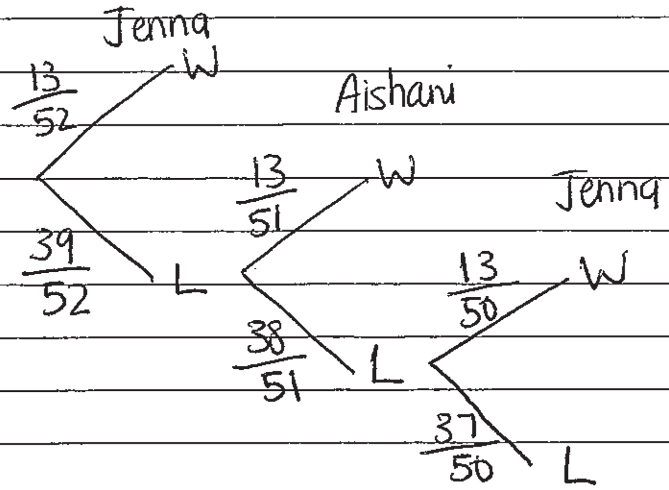
$x = -1$ is the only solution.

- 1 mark for solving for one case correctly
- 2 marks for solving for two cases correctly
- 3 mark for testing correct answers to show only $x = -1$ is the solution

d) i) P(J win on first pick) = $\frac{13}{52} = \frac{1}{4}$

1 mark for answer

ii)



P(J wins on second pick) $= \frac{39}{52} \times \frac{38}{51} \times \frac{13}{50}$

$= \frac{247}{1700}$

• 1 mark if two correct outcomes listed • 2 marks if correct answer

e) $y = \frac{3x}{2} + 2$ and $y = \frac{3x}{2} - 4$

Distance between two points = 6

Locus is 3 units from other lines

y-int of locus = −1

locus equation: $y = \frac{3x}{2} - 1$ OR $3x - 2y - 2 = 0$

• 1 mark for working towards answer (finding distance) • 2 marks for correctly finding locus

Question 13 Feedback

a) Mostly done well. Easiest way to complete this question is to draw a diagram - some people didn't do this.

b) i) Some people substituted (2, 5) to show it was a point on the line. This was not a valid method as they did not show (2, 5) was the point B. Students needed to solve the equations simultaneously to find the point of intersection.
- Other common error was not finding y.

i) Some people used the formula incorrectly - it is on the formula sheet so students just needed to copy it correctly.
- Some found the wrong equation of AC
- Some had the equation with fractions in it, which made it more difficult for them to substitute values into the formula

ii) Some students didn't use the perpendicular distance to find the area.
- Students needed to simplify their answer.

c) Most common error was not testing solutions.

i) Mostly correct.
i) Most common error was not including Aishani losing on her first pick in the calculation. It's best if students draw a tree diagram.

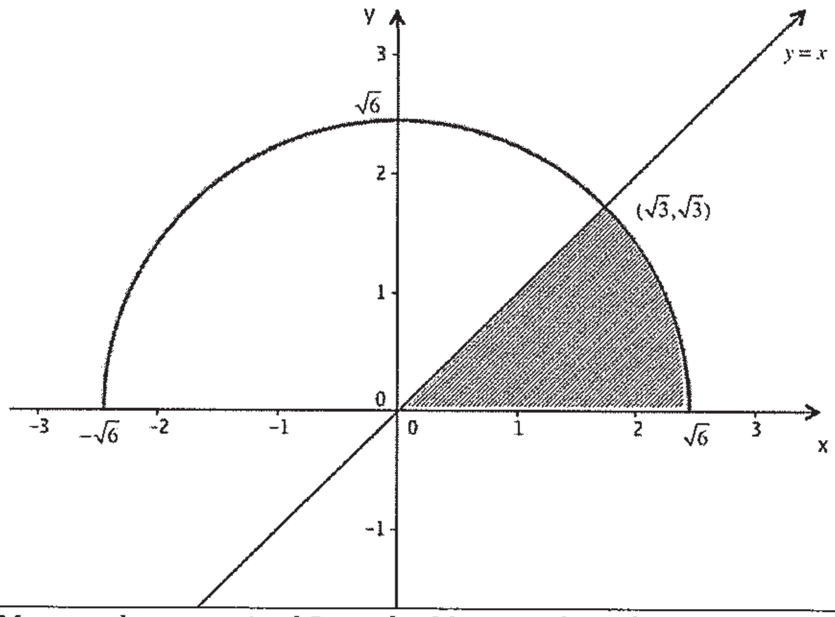
e) A number of errors were made with this question.
- Some tried to find the point of intersection between the parallel lines
- Some had an answer with no working or justification.
- Some tried to find the midpoint, but subtracted instead of adding.
- Others had no idea how to attempt it.

Question 14

(a) Sketch the region on the Cartesian plane for which the following inequalities hold true, showing all important features. **3**

$$y \le x$$
$$y \ge 0$$
$$y \le \sqrt{6 - x^2}$$

3 marks sketch correct region with intersection point and intercepts shown
2 marks sketches correct region, intersection point or intercepts missing
Correct sketch of semicircle with intercepts or Correct region and no intercepts or intersection
1 mark deducted for each of – no intersection point, no intercepts, incorrect solid/dashed line



Most students received 2 marks. Many students lost 1 mark for not showing the intercept. Students also lost a mark if intercepts were not shown. Common error was showing the incorrect radius when the semi-circle was plotted. There was no deduction for not testing a point.

(b) Use Simpsons Rule with 3 function values to evaluate $\int_0^{\frac{\pi}{4}} \sec x \, dx$ **3**

Give your answer correct to 2 decimal places .

3 marks correct answer all working shown
2 marks correct "h", correct use of Simpsons rule or correct values, correct "h"
1 mark correct value of "h" only

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$
$\sec x$	1	$\sec\frac{\pi}{8}$	$\sqrt{2}$

$$\int \sec x \, dx \approx \frac{\pi/8}{3}\left[1+4\sec\frac{\pi}{8}+\sqrt{2}\right]$$

$=0.8827...$

$=0.88$ (2dp)

Done well. Most students received 2-3 marks. Students that drew a table were more successful. Some students had difficulty with the "sec" function. Other students used more than 3 function values.

(c)

2 marks correct expression with constant evaluated
1 mark correct expression no constant evaluated

(i)

$x=\int v \, dt$

$=4t-2\ln(t+1)+c$

initial conditions $t=0, x=0$

$x=4t-2\ln(x+1)$

Done well. Common error was incorrect integral of $\int \frac{1}{t+1} dt = \ln(t+1)+c$

(ii)

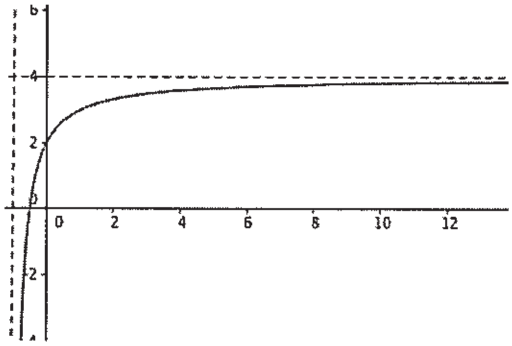
1 mark valid explanation

$$v = 4 - \frac{2}{t+1}$$

when $v = 4 \Rightarrow 4 = 4 - \frac{2}{t+1}$

$\therefore \frac{2}{t+1} = 0$ which is not possible

OR sketch and explanation



Done well. Mark was given for any valid explanation. Time can't be negative was not accepted as this does not relate to v=4

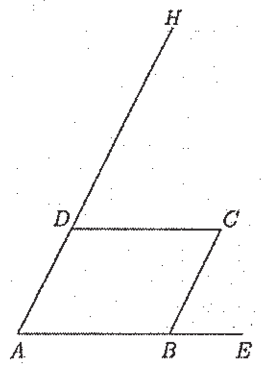
(iii)

2 marks correct derivative and value when t=2
1 mark, correct derivative

$$v = 4 - 2(t+1)^{-1}$$
$$a = v' = 2(t+1)^{-2}$$
$$= \frac{2}{(t+1)^2}$$
$$= \frac{2}{9} \text{ when } t = 2$$

Done reasonably well. Several students made this hard by using the quotient rule.

(d) In a parallelogram $ABCD$ the sides AB and AD are extended to E and H respectively so that $\frac{AD}{DH} = \frac{BE}{AB}$.



Copy or trace the diagram into your writing booklet .

i. Prove that ΔHDC and ΔCBE are similar. 3

ii. Hence or otherwise show that the points H, C and E lie on a straight line . 1

3 marks corrects complete proof with reasoning
2 mark correct expression no constant evaluated. Partly completed proof, 1 mark deducted each error
1 mark 1 correct angle with reasoning

$AD \parallel BC$ (opposite sides of a parallelogram are parallel)

$\therefore HD \parallel BC$ (ADH lie on the same line, given)

$DC \parallel AB$ (opposite sides of a parallelogram are parallel)

$\therefore BE \parallel DC$ (ABE lie on the same line, given)

$AD = BC$ (opposite sides of a parallelogram are equal)

AB=DC (opposite sides of a parallelogram are equal)

$\frac{AD}{DH} = \frac{BE}{AB}(Given)$

$\therefore \frac{BC}{DH} = \frac{BE}{DC}$

In ΔHDC and ΔCBE

$\angle HDC = \angle CBE$ (corresponding angles, AH $\parallel CB$, $DC \parallel AE$)

$\frac{BC}{DH} = \frac{BE}{DC}$(shown above)

$\therefore \Delta HDC \;|||\; \Delta CBE$ (two corresponding sides in proportion and included angle equal)

An equiangular proof is not valid as line HE is not given, it cannot be assumed to be straight. Students had to use the reason that "two corresponding sides are in proportion and included angle is equal". Students lost a mark for not stating why lines were parallel (opposite sides of a parallelogram are parallel). A mark was deducted for using abbreviations in the conclusion of he proof. This is usually only accepted in Congruence. Take care to name angles correctly and use full complete reasoning.

1 mark valid proof with reasoning

(ii)
$DC \parallel BE$ (shown above)

$\angle HCD = \angle CEB$ (corresponding angles in similar triangles are equal)

point C is common

$\therefore HCE$ lie on the same line

Some long proofs considering it was only 1 mark.

(a) (i) Given $y = 1 - 12x + x^3$

$\frac{dy}{dx} = -12x + 3x^2$ $\qquad$ $\frac{d^2y}{dx^2} = 6x.$

$= 3(x^2 - 4)$

hence $\frac{dy}{dx} = 0$, $x = \pm 2$ $\qquad$ ① for finding the value of x.

when $x = 2$, $y = -15$

$\frac{d^2y}{dx^2} = 12 > 0$ $\qquad$ ① for determining their nature

when $x = -2$, $y = 17$

$\frac{d^2y}{dx^2} = -12 < 0$

∴ stationary points are $(2, -15)$ minimum turning point $\qquad$ ① for stationary pt

$(-2, 17)$ maximum turning point.

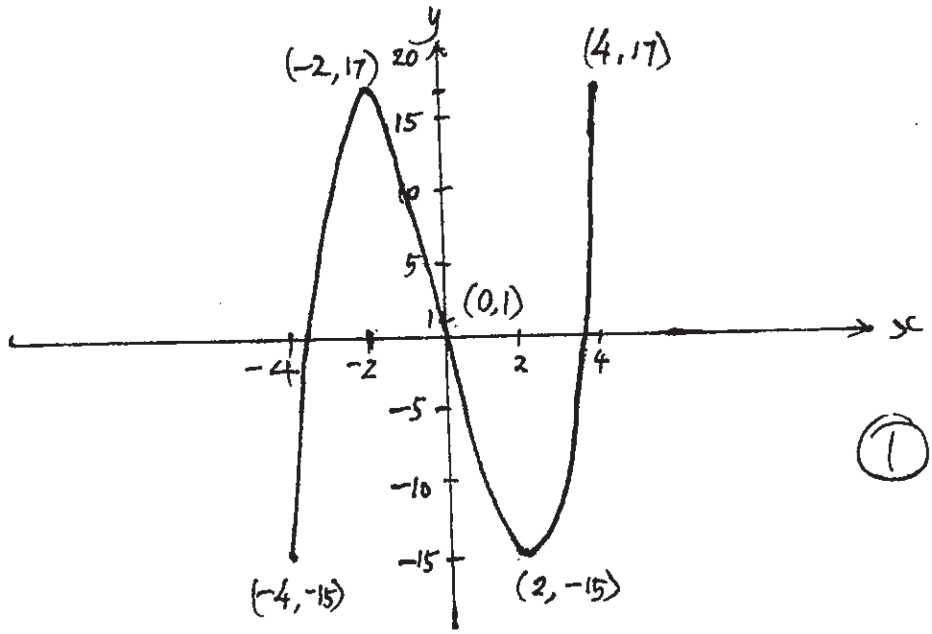
(ii) Point of inflexion at $\frac{d^2y}{dx^2} = 0$, $\quad 6x = 0$

$x = 0$, when $x = 0$, $y = 1$ $\qquad$ ① for testing

x	-1	0	1
$\frac{d^2y}{dx^2}$	-6	0	6

∴ $(0, 1)$ is the point of inflexion. $\qquad$ ① for point of inflexion

(iii)



① for sketching the curve.

☐ ⇦ Tick this box if you have continued this answer in another writing booklet.

Office Use Only – Do NOT write anything, or make any marks, below this line.

(b) (i) $y=\sqrt{9-x^2}$

$\frac{dy}{dx}=\frac{1}{2}(9-x^2)^{-\frac{1}{2}}(-2x)$ ① for differentiating the function

$=\frac{-x}{\sqrt{9-x^2}}$ ① for answer

(ii) $\int \frac{6x}{\sqrt{9-x^2}}\,dx = \int -6\frac{dy}{dx}\,dx$ ① for integrating the fn.

$= -6y + c$ since $y=\sqrt{9-x^2}$

$\therefore = -6\sqrt{9-x^2}+c$ ① for answer

(c) $V=\pi\int_0^1 y^2\,dx = \pi\int_0^1\left(\frac{1}{\sqrt{2x+1}}\right)^2 dx$

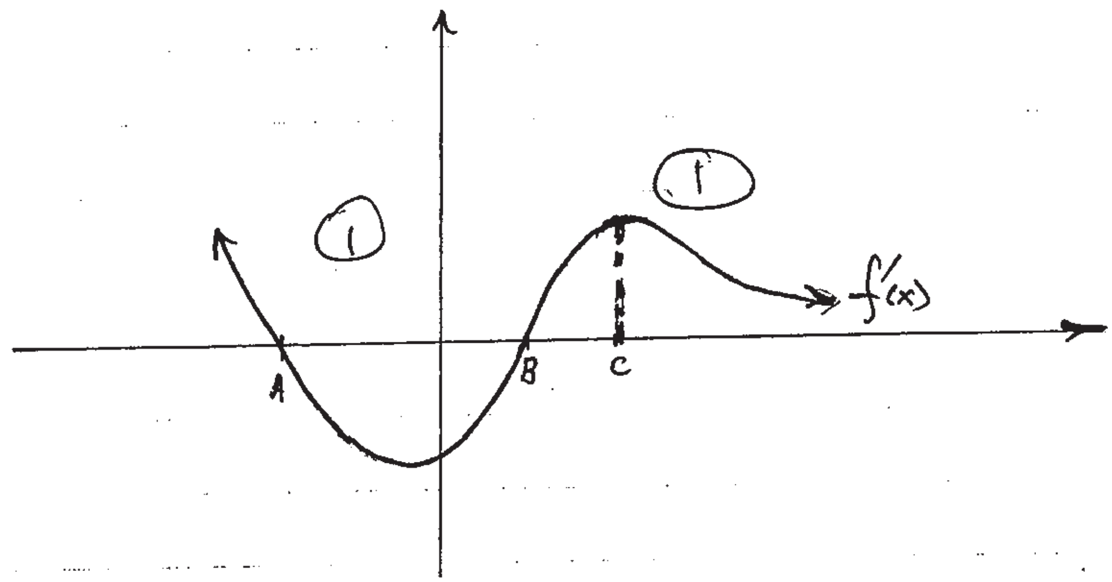
$=\pi\int_0^1 \frac{1}{2x+1}\,dx$ ① for substituting the function

$=\frac{\pi}{2}\int_0^1 \frac{2}{2x+1}\,dx$ ① for integrating the function

$=\frac{\pi}{2}\left[\ln(2x+1)\right]_0^1$

$=\frac{\pi}{2}\ln 3$ units^3 ① for answer.

(d)



15 (a) Students have done very well with stationary points, determine their nature, point of inflexion and sketch the curve.

(b) Approximate a quarter of students forgot to put negative sign when differentiate and same for integration of a function.

(c) When finding the exact volume of the solid students forget to square the function and later to halve the π when differentiate the function.

(d) Students have done well for sketching the graph of $f'(x)$.

2 unit

Q16 a

i $Q = Ae^{-kt}$

$\left(\frac{5}{8}A = Ae^{-10k}\right) \div A$ (1 mark)

$\frac{5}{8} = e^{-10k}$

$\ln\left(\frac{5}{8}\right) = \ln\left(e^{-10k}\right)$

$-10k = \ln\left(\frac{5}{8}\right)$

$k = \frac{\ln\left(\frac{5}{8}\right)}{-10}$

$\therefore k = 0.0470036292$ (1 mark)

$\therefore Q = Ae^{-0.047\ldots t}$

when $t = 20$ $Q = A \cdot e^{-0.047\ldots \times 20}$

$Q = 0.390625A$

$\therefore Q = \frac{25}{64}A$ (1 mark)

$\therefore \frac{25}{64}$ of fuel left in the tank

also paid $\frac{39.06}{100}$ and $\frac{1}{e^{0.94}}$

ii $A = 60$

$\frac{dQ}{dt} = -kQ$

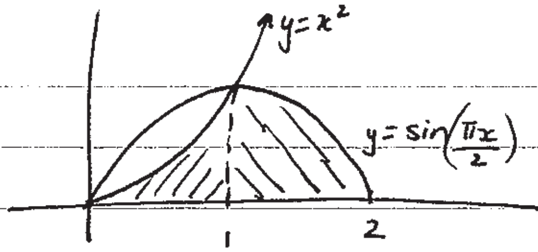
$= -0.047\ldots \times \frac{25}{64} \times 60$

$= -1.10151006$

$= -1.1$ L/min (to 1 dec place) (1 mark) ECF given

$\therefore$ flowing out at 1.1 L/min

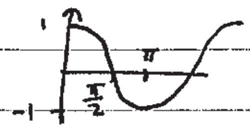
16b i



$$A = \int_0^1 x^2\,dx + \int_1^2 \sin\left(\frac{\pi x}{2}\right) dx$$ (1 mark)

$$= \left[\frac{x^3}{3}\right]_0^1 + \left[\frac{-2}{\pi}\cos\left(\frac{\pi}{2}x\right)\right]_1^2$$

$$= \left(\frac{1}{3} - 0\right) \; \frac{-2}{\pi}\left[\cos(\pi) - \cos\left(\frac{\pi}{2}\right)\right]$$



Note: 1 mark awarded if correct integration

$$= \frac{1}{3} - \frac{2}{\pi}(-1)$$

$$= \frac{1}{3} + \frac{2}{\pi} \text{ units}^2$$ (1 mark)

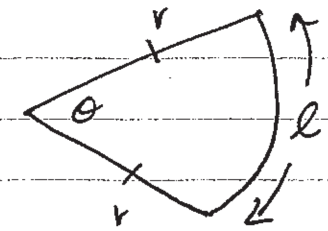
---

ii $V = \pi \int y^2\,dx$

$$V = \pi\left[\int_0^1 x^4\,dx + \int_1^2 \sin^2\left(\frac{\pi x}{2}\right) dx\right]$$

16c

i



$P = 8$

$l = r\theta$

$\therefore\ 8 = 2r + r\theta$

$8 = r(2+\theta)$

$r = \frac{8}{2+\theta}$

$A = \frac{1}{2} r^2 \theta$

$A = \frac{1}{2}\left(\frac{8}{2+\theta}\right)^2 \times \theta$

$A = \frac{64\theta}{2(2+\theta)^2}$

$A = \frac{32\theta}{(2+\theta)^2}$

ii

$$A = \frac{32\theta}{(2+\theta)^2}$$

$$y = \frac{u}{v}$$

$$y' = \frac{u'v - v'u}{v^2}$$

$u = 32\theta$

$u' = 32$

$v = (2+\theta)^2$

$v' = 2(2+\theta)$

$$\frac{dA}{d\theta} = \frac{u'v - v'u}{v^2}$$

$$= \frac{32(2+\theta)^2 - 2(2+\theta)32\theta}{(2+\theta)^4}$$

$$= \frac{\cancel{(2+\theta)}[32(2+\theta) - 64\theta]}{(2+\theta)^{\cancel{4}\,3}}$$

$$= \frac{(64 + 32\theta - 64\theta)}{(2+\theta)^3}$$

$$\frac{dA}{d\theta} = \frac{(64 - 32\theta)}{(2+\theta)^3}$$

let $\frac{dA}{d\theta} = 0$

$\therefore 64 - 32\theta = 0$

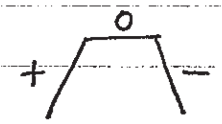
$32\theta = 64$

$\therefore \theta = 2$ (1 mark)

Prove a max

θ	1	2	3
$\frac{dA}{d\theta}$	$\frac{32}{27}$	0	$\frac{-32}{125}$

(1 mark)

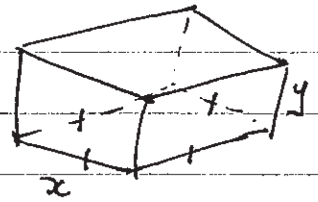


$\therefore$ max when $\theta = 2$

$$\therefore A = \frac{32\theta}{(2+\theta)^2} = \frac{32\times 2}{(2+2)^2} = \frac{64}{16} = 4m^2$$

(1 mark)

16d
i



$V = x^2 y$

$2 = x^2 y$

$\therefore y = \dfrac{2}{x^2}$

S. Area of the Box = Base + 4 equal sides

$A = x^2 + 4 \times \cancel{x} \times \dfrac{2}{x^{\cancel{2}}}$

$A = x^2 + \dfrac{8}{x}$

ii Prove a min. if $x = 2y$

$A = x^2 + 8 . x^{-1}$

$\dfrac{dA}{dx} = 2x - 8x^{-2}$

let $\dfrac{dA}{dx} = 0$

$0 = 2x - \dfrac{8}{x^2}$

$\dfrac{8}{x^2} = 2x$

$8 = 2x^3$

$\therefore x^3 = 4$

(1 mark) $\therefore x = \pm \sqrt[3]{4}$

$\therefore x = \sqrt[3]{4}$ as x is a length $\therefore x > 0$

Prove a min

$\dfrac{dA}{dx} = 2x - 8x^{-2}$

$\dfrac{d^2A}{dx^2} = 2 + \dfrac{16}{x^3}$ sub. $x = \sqrt[3]{4}$ ECF (1 mark)

$= 2 + \dfrac{16}{{}^3(\sqrt[3]{4})} = 2 + 4 = 6$ as $f''(x) > 0$ $\therefore$ a min.

as $y = \frac{2}{x^2}$

sub $x = \sqrt[3]{4}$

$$y = \frac{2}{(\sqrt[3]{4})^2}$$

$$y = \frac{2}{4^{2/3}}$$

as $4 = 2^2$

$$y = \frac{2^1}{2^{4/3}}$$

$\therefore y = 2^{-1/3}$

if $x = 2y$

$$y = \frac{x}{2}$$

$$y = \frac{\sqrt[3]{4}}{2}$$

$$y = \frac{4^{1/3}}{2}$$

$$y = \frac{2^{2/3}}{2^1}$$

$y = 2^{-1/3}$ (1 mark)

$\therefore$ a min when $x = 2y$
as answer is the same as sub.
min value into $y = \frac{2}{x^2}$

→ other techniques used by students were awarded marks for reaching similar points in their working.

Markers Feeback 2U Trial Q16

a) i) Exponential decay - done well by most students.
Some had calculator issues and lost the last mark.

ii) many didn't apply $\frac{dQ}{dt} = -kQ$. About half did and got the mark.

→ Some students left all of part a) blank, and need to heavily revise this content.

b) i) Area → more than half of the cohort got this wrong. these students subtracted instead of splitting the areas. - many didn't get an ECF mark for correct integration as they had trouble integrating $\sin(\frac{\pi x}{2})$.

ii) about 3/4 of students got this wrong. each volume needs to be done separately.

c) i) most proved this well if they saw
$8 = 2r + l$, where $l = r\theta$, which converted to
$r = \frac{8}{2+\theta}$.

ii) many did this well. However, about 40% had issues with the Quotient Rule as they didn't factorise out $(\theta+2)$ in the numerator. Some didn't prove the 'maximum' correctly, and didn't show Area $= 4m^2$

d) i) many did this well, those who didn't were unable to show $A = x^2 + 4xy$

ii) There were over 8 ways students did this question gaining full marks, which was very pleasing.
Some students didn't prove a 'minimum', or failed to attempt the question. Many would benefit from practicing maxima/minima questions...